A Queueing Network Model with Feedback and its Application in Healthcare

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Abstract: In recent years application of queueing systems and networks in the healthcare area arouses researchers' interest. This paper describes how system dynamics was used as a central part of a complex system of fertility healthcare to improve its performance. We consider a healthcare system of infertility clinic as an open queueing network, where the patients/couples arrive at the clinic according to a homogeneous Poisson process and get self-service with a general service time distribution for completing an initial non-clinical registration formality. Then the couples directly enter the first node of the system. After completing service at the first node, depending on the current status of the couples, they are routed either to the second or third nodes with probabilities p_1 and $1-p_1$ respectively. After completion of service at the second node couples can either leave the system or enter the third node with probability p_2 and $1-p_2$ respectively. From the third node couples either get readmitted (immediate feedback) with probability p_3 for further treatments at the same node or leave the network after treatment completion with probability p_4 or get rerouted (delayed feedback) to the second node with probability $1-p_3-p_4$. All the nodes follow $M|M|1|\infty|FCFS$ schedule. We formulate a network model with feedback flows for this clinical situation and obtain the steady state solution. Some key performance measures are calculated for this system and a numerical illustration provided.

Keywords: Feedback, performance measures, queueing network, steady state.

I. INTRODUCTION

Queueing systems and queueing networks are successfully used for performance analysis of different systems such as computer networks, telecommunication systems, call centers, flexible manufacturing systems and similar service systems. A considerable body of research has shown that queueing theory can be useful in real world healthcare situations. Many countries of the world have seen fertility rate declines. Sincere efforts have been made by the government to change this trend. Understanding the dynamics of fertility treatment systems through queueing models can contribute to the possible improvement of the fertility treatment system.

Queues are formed when entities that demand service, usually refered to as customers', arrive at a service facility' and cannot be served immediately upon arrival. In healthcare delivery systems, patients are typically the customers and either outpatient clinics or hospitals or diagnostic imaging centers are the service facilities. A service facility may consists of one or more service stations where customers are served. Following Bose [2], a queueing network is a system of interconnected queues providing service. Queueing networks can be classified as open, closed and mixed with respect to the number of customers in the network. In an open network customers enter from outside, receive service at systems and leave the network. In closed network, a constant number of customers continually circulate in the network with no other arrival to the system and departures from the system. Depending on the number of customer classes we have single or multi class networks. In the case of mixed network, the network may be open for some classes of customers and closed for some other classes. Queueing network models have various applications in many areas, such as service centers, computer networks, communication networks, production and flexible manufacturing systems, airport terminals and healthcare systems etc.

In this paper we are modeling a healthcare situation as an open queueing network. Healthcare systems in general, and hospitals in particular, constitute a very important part of the service sector. Over the years, hospitals have become increasingly successful in deploying medical and technical innovations to more effective clinical treatments. However, they are still often rife with inefficiencies and delays, thus presenting a propitious ground for research in numerous scientific fields, specifically queueing theory. Of particular interest to queueing scientists is the topic of patient flows in hospitals; improving it can have a significant impact on quality of care as well as on patient satisfaction. Hence patient flow management is an important topic of the present time and present itself with a host of challenging and important research questions. Obviously, patient flow has caught the attention of researchers in operations research, applied probability, service engineering and operations management; with queueing theory being a common central theme on all these disciplines. The reason is that hospitals experience frequent congestion which result in significant delays. Hence they fit naturally under the framework of queueing theory, which addresses the trade-offs between operational service quality versus resource efficiency. Our attempt here is to stimulate relevant research, with ultimate goal being delay reduction with its accompanying important benefits: clinical, financial, psychological and societal. Healthcare systems offer many complex problems that could benefit from operations research type analysis and applications (see Carter[4]). The long waiting times of patients, the lack of resources and offload delay problems facing the medical services are among these issues.

Our starting point is that a queueing network encapsulates the operational dimensions of patient flows in hospitals, with medical units being the nodes of network, patients are the customers, while beds, medical staff and medical equipments are the servers. One need to consider the special features of this queueing network in terms of the system primitives, key performance measures and available control mechanisms. Through the extensive data set of patient flow based on the queueing network model, we may get more insight on the whole system. Queueing models have a long tradition of being useful tools for evaluating the performance of healthcare systems in which waiting lists occur, ever since the work of Bailey [1]. For instance Jackson et al. [12], Worthington [18] and Goddard et al. [5] modeled appointment systems and waiting list management in outpatient clinics. Patients flow in hospitals has been extensively studied by many researchers. One may refer to the work of Hall [6,7] for the methodological advancement of patient flow analysis in healthcare. Roumani [16] suggests a queueing network model of ICUs using discrete event simulation, provides network model for patient flow considering both instantaneous and delayed feedback. The most significant contribution in queueing network is Jackson's network (Jackson [10,11]) wherein the joint state distribution for the system can be written as the product of individual state probabilities. Using Jackson's theorem, if n_i is the number of customers at node S_i having traffic intensities ρ_i (i=1,2,…,k.) then

$$P[S_1=n_1,S_2=n_2,\ldots,S_k=n_k]=(1-\rho_1)\rho_1^{n_1}(1-\rho_2)(\rho_2)^{n_2}\ldots(1-\rho_k)(\rho_k)^{n_k}. \quad (1)$$

Indeed it is applicable for an arbitrary network of $M|M|m|\infty$ queueing nodes, where jobs arrive in Poisson stream from outside (to one or more stations) and are routed to one node to another until they eventually depart from the system. In the literature there are many attempts to represent a health care system as network of queues. But many of them are analyzed using computer simulation. Taylor and

Keown [17] analyze a system model for Burncare. Harper and Shahani [8] suggest a simulation model in the planning and management of bed capacities in hospitals. Hershey et al. [9] suggest a methodology for estimating expected utilization and service level for a class of capacity constrained service network facilities operating in a stochastic environment. Most of the literature in the healthcare system deal with single station queueing systems. Using the result of Mirasol [14] we have that for a queueing system with a Poisson input and an infinite number of channels the output is also Poisson regardless of the distribution of service times, provided these times are independent. Motivated by the application of queueing network in a healthcare system of an infertility clinic we investigate a model for an arbitrary open queueing network consisting of three $M|M|1$ nodes with immediate and delayed feedback and propose its steady state solution.

The paper is organized as follows. Section 2 briefly describes the structure of the network system and the routing pattern of the customers. Section 3 presents the network model of the service system and the steady state solution. In section 4 several performance parameters of the service system are developed and then followed by a numerical experiment in the succeeding section.

II. STRUCTURE AND ROUTING PATTERNS OF THE SYSTEM

We consider an infertility clinic as an open queueing network system, where the patients/couples arrive at the clinic according to a homogeneous Poisson process and get self-service with a general service time distribution for completing an initial non-clinical registration formality. According to the revised glossary of the International Committee for Monitoring Assisted Reproductive Technology (ICMART) and the World Health Organization (WHO) the clinical definition of infertility is a disease of the reproductive system defined by the failure to achieve a clinical pregnancy after twelve months or more of regular unprotected sexual intercourse " (see Zegers-Hochschild et.al [19]). For the treatment of this disease the male and female partners are evaluated for infertility or subfertility using a variety of clinical interventions and also from certain laboratory evaluations on the individuals. At the initial registration facility, specific enquiries about lifestyle and sexual history of couples are collected to identify people who are less likely to conceive. After completing this formality they enter into the system, that is the first node which follows an $M|M|1|\infty|FCFS$ schedule. Here the forms are checked by the server and the people are divided into two categories according to years after marriage'. Clinic uses this as the initial predictor and does not check the other details in the first node. Less than one year after marriage' (<1) couples routed to the second node with probability p_1. More than one year after marriage' couples can directly enter the third node for treatment with probability $1-p_1$. In the former case, the details of the couples are checked and they undergo a counseling process. This node also follows an $M|M|1|\infty|FCFS$ schedule. After the counseling session, the couples who realized that they don't require treatment leave the system with probability p_2. If they want treatment, directed to the third node with probability $1-p_2$. This third node follows an $M|M|1|\infty|FCFS$ schedule. Here couples are treated and they leave the system with probability p_3. This type of leaving may be due to completion of treatment (such as one day treatment) or giving up of treatment. Some couples want more cycles of treatments they re-enter the same node (immediate feedback) with probability p_4. Some may be mentally stressed after consultation and want counseling and so can enter the second node with probability $1-p_3-p_4$ (delayed feedback). After counseling those couples who are convinced of their problems may leave the system with probability p_2. Otherwise they continue the treatment with probability $1-p_2$. That is re-enter the third node. The queueing network envisaged here is that of three Markovian queueing models with a Bernoulli schedule for routing and with feedbacks.

III. MODELING THE SERVICE SYSTEM

We consider an open queueing network consisting of three single server nodes (service centers) and Q_i denote the queue corresponding to the i^{th} node where i=1,2,3. Customers arrive to the system according to a homogeneous Poisson process (external arrival) and get self service with a general service time distribution. After that customers are entered to the first node with rate λ. We assume that the service times are exponentially distributed with service rates μ_1, μ_2 and μ_3 at the respective nodes. We also assume that there is an infinite waiting space capacity for each node, a single class of patients and that patients served on FCFS discipline. The routing probabilities in the network are assumed to be p_1 from node one to node two, p_2 is the leaving probability from node two and $1-p_2$ is the transfer probability from node two to node three. p_3 is the leaving probability from node three. Third node has immediate feedback and delayed feedback with probability p_4 and $1-p_3-p_4$ respectively. Both types of feedbacks are assumed to be independent of the state of the system and any previous feedback. The state of the system can be represented by a three state Markov process $\{N_1(t),N_2(t),N_3(t),t\geq 0\}$ where $N_i(t)$ represents the number of couples present in the node i' at time t' (i=1,2,3). Our objective is to find the steady state solution having n_1 patients at node one, n_2 patients at node two and n_3 patients at node three. It is denoted by P_{n_1,n_2,n_3}. A diagrammatic representation of the above healthcare queueing network is shown in Figure 1.

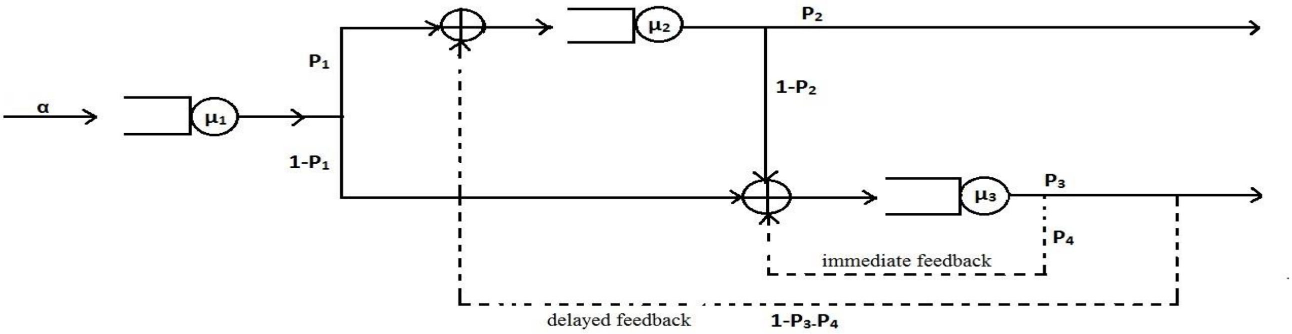

Figure 1 : Health care queueing network with 3 nodes and 2 feedbacks

Following the standard procedure of formulating differential-difference equation satisfied by the transient probabilities of the system and then deducing the balance equations one can obtain the steady state solution of the model.

Using the local balance approach, we define λ_i where (i=1,2,3) is the average total arrival rate to each node as follows :

For node 1 : $\lambda_1=\lambda$. (2)

For node 2 : $\lambda_2=\lambda p_1+\lambda_3(1-p_3-p_4)$. (3)

For node 3 : $\lambda_3=\lambda(1-p_1)+\lambda_2(1-p_2)+\lambda_3 p_4$. (4)

Solving these systems of equations, we get

$$\lambda_3=\frac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}, \tag{5}$$

and

$$\lambda_2=\frac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}. \tag{6}$$

Using mathematical technology of Jackson network if n_1,n_2,n_3 are number of customers at each nodes therefore probability of overall system,

$$P(n_1,n_2,n_3)=(1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{n_2}(1-\rho_3)\rho_3^{n_3}. \tag{7}$$

Where $\rho_1=\frac{\lambda_1}{\mu_1}$, $\rho_2=\frac{\lambda_2}{\mu_2}$ and $\rho_3=\frac{\lambda_3}{\mu_3}$.

According to the above result, we get the following steady state solution for queueing network model with instantaneous and delayed feedback.

$$\begin{aligned}P(n_1,n_2,n_3) &= \left(\frac{\lambda_1}{\mu_1}\right)^{n_1}\left(1-\frac{\lambda_1}{\mu_1}\right)*\left(\frac{\lambda_2}{\mu_2}\right)^{n_2}\left(1-\frac{\lambda_2}{\mu_2}\right)*\left(\frac{\lambda_3}{\mu_3}\right)^{n_3}\left(1-\frac{\lambda_3}{\mu_3}\right)\\ &= \left(\frac{\lambda}{\mu_1}\right)^{n_1}\left(1-\frac{\lambda}{\mu_1}\right) * \left[\frac{\lambda(1-p_3-p_4+p_1p_2p_4)}{\mu_2(p_2(1-p_3-p_4)+p_4)}\right]^{n_2}\left[1-\frac{\lambda(1-p_3-p_4+p_1p_2p_4)}{\mu_2(p_2(1-p_3-p_4)+p_4)}\right] *\\ &\quad \left[\frac{\lambda(1-p_1p_2)}{\mu_3(p_2(1-p_3-p_4)+p_4)}\right]^{n_3}\left[1-\frac{\lambda(1-p_1p_2)}{\mu_3(p_2(1-p_3-p_4)+p_4)}\right]\end{aligned} \tag{8}$$

IV. NETWORK PERFORMNCE PARAMETERS

Jackson's theorem (see Bose [2], page - 152) is used to obtain the state distribution of each queue in isolation and overall state distribution as the product of the individual distributions. Using these flows and the state distributions the following network performance parameters are calculated.

We know that throughput Λ_i is the average arrival rate from a Poisson process from outside entering the network to the i^{th} queue Q_i where (i=1,2,3). Here the customers can enter the network at one point only after completing a formal registration procedure with rate λ. Hence the total throughput is the total external arrival rate, which is equal to λ.

A. *Average Visit Count to Nodes*

Visit count to a node i is a measure of the average number of visits a couple makes to the queue Q_i where (i=1,2,3) once it enters the queueing network. Let V_i be the visit count to node i (i=1,2,3). Then we get

$$\begin{aligned} V_1 &= \frac{\lambda_1}{total \quad throughput} \\ &= \frac{\lambda}{\lambda} \\ &= 1. \end{aligned} \tag{9}$$

Note that V_1 ought to be one since every couple definitely visit node one. Now

$$\begin{aligned} V_2 &= \frac{\lambda_2}{total \quad throughput} \\ &= \frac{(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}. \end{aligned} \tag{10}$$

and

$$\begin{aligned} V_3 &= \frac{\lambda_3}{total \quad throughput} \\ &= \frac{(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}. \end{aligned} \tag{11}$$

B. *Average Number of Couples in the System*

Using the results of $M|M|1$ queues we find the average number of couples in the three queues. Let $N_i(i=1,2,3)$ be the average number of couples in $Q_i(i=1,2,3)$.
Then we have

$$\begin{aligned} N_1 &= \frac{\rho_1}{1-\rho_1} \\ &= \frac{\lambda}{\mu_1-\lambda}. \end{aligned} \tag{12}$$

$$\begin{aligned} N_2 &= \frac{\rho_2}{1-\rho_2} \\ &= \frac{\dfrac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}{\mu_2-\dfrac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}. \end{aligned} \tag{13}$$

$$N_3 = \frac{\rho_3}{1-\rho_3}$$

$$= \frac{\dfrac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}{\mu_3-\dfrac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}. \tag{14}$$

Hence the average number of couples in the overall system, N is obtained by

$$N = N_1+N_2+N_3$$

$$= \frac{\lambda}{\mu_1-\lambda}+\frac{\dfrac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}{\mu_2-\dfrac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}+\frac{\dfrac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}{\mu_3-\dfrac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}. \tag{15}$$

C. Average Waiting Time of a Couple

Average waiting / sojourn time of a couple means the total time spent in the system by an arriving couple before they leave the network. This may be found either as the sum of the total times that the couple will spend in each of the three queues in the network or may apply Little's result directly to the overall network. If W_i denotes the average waiting time of a couple at node i (i=1,2,3) then we have

$$W_1 = \frac{V_1N_1}{\lambda_1} = \frac{N_1}{\lambda}. \tag{16}$$

$$W_2 = \frac{V_2N_2}{\lambda_2} = \frac{N_2}{\lambda}. \tag{17}$$

$$W_3 = \frac{V_3N_3}{\lambda_3} = \frac{N_3}{\lambda}. \tag{18}$$

Hence the average waiting time of a couple in the system, W is

$$W = W_1+W_2+W_3 = \frac{1}{\lambda}(N_1+N_2+N_3)$$

$$= \frac{1}{\mu_1-\lambda}+\frac{\dfrac{(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}{\mu_2-\dfrac{\lambda(1-p_3-p_4+p_1p_2p_4)}{p_2(1-p_3-p_4)+p_4}}+\frac{\dfrac{(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}{\mu_3-\dfrac{\lambda(1-p_1p_2)}{p_2(1-p_3-p_4)+p_4}}. \tag{19}$$

V. NUMERICAL EXAMPLE

A Matlab program has been developed for finding the steady state solution and the values of performance parameters. The steady state probabilities and the performance measures are computed for the following parametric values are given in Table I and Table II.

$\lambda=0.1, p_1=0.4, p_2=0.2, p_3=0.5, p_4=0.3, \mu_1=0.6, \mu_2=1.1, \mu_3=2.1$

Table I: Performance measures

V_1	V_2	V_3	N_1	N_2	N_3	N	W
1	0.6588	2.7059	0.2	0.0637	0.1479	0.4116	4.1162

Table II: Steady state distribution

$\lambda=0.1, p_1=0.4, p_2=0.2, p_3=0.5, p_4=0.3, \mu_1=0.6, \mu_2=1.1, \mu_3=2.1$

(n_1,n_2,n_3)	$P(n_1,n_2,n_3)$	(n_1,n_2,n_3)	$P(n_1,n_2,n_3)$	(n_1,n_2,n_3)	$P(n_1,n_2,n_3)$
0 0 0	6.82E-01	1 3 2	4.06E-07	3 1 4	5.22E-08
0 0 1	8.79E-02	1 3 3	5.23E-08	3 2 0	1.13E-05
0 0 2	1.13E-02	1 3 4	6.74E-09	3 2 1	1.46E-06
0 0 3	1.46E-03	1 4 0	1.46E-06	3 2 2	1.88E-07
0 0 4	1.88E-04	1 4 1	1.89E-07	3 2 3	2.42E-08
0 1 0	4.09E-02	1 4 2	2.43E-08	3 2 4	3.12E-09
0 1 1	5.27E-03	1 4 3	3.13E-09	3 3 0	6.79E-07
0 1 2	6.79E-04	1 4 4	4.03E-10	3 3 1	8.75E-08
0 1 3	8.74E-05	2 0 0	1.90E-02	3 3 2	1.13E-08
0 1 4	1.13E-05	2 0 1	2.44E-03	3 3 3	1.45E-09
0 2 0	2.45E-03	2 0 2	3.15E-04	3 3 4	1.87E-10
0 2 1	3.15E-04	2 0 3	4.06E-05	3 4 0	4.07E-08
0 2 2	4.06E-05	2 0 4	5.23E-06	3 4 1	5.24E-09
0 2 3	5.24E-06	2 1 0	1.14E-03	3 4 2	6.75E-10
0 2 4	6.75E-07	2 1 1	1.46E-04	3 4 3	8.70E-11
0 3 0	1.47E-04	2 1 2	1.89E-05	3 4 4	1.12E-11
0 3 1	1.89E-05	2 1 3	2.43E-06	4 0 0	5.27E-04
0 3 2	2.43E-06	2 1 4	3.13E-07	4 0 1	6.79E-05
0 3 3	3.14E-07	2 2 0	6.80E-05	4 0 2	8.74E-06
0 3 4	4.04E-08	2 2 1	8.76E-06	4 0 3	1.13E-06
0 4 0	8.78E-06	2 2 2	1.13E-06	4 0 4	1.45E-07
0 4 1	1.13E-06	2 2 3	1.45E-07	4 1 0	3.15E-05
0 4 2	1.46E-07	2 2 4	1.87E-08	4 1 1	4.06E-06
0 4 3	1.88E-08	2 3 0	4.07E-06	4 1 2	5.24E-07
0 4 4	2.42E-09	2 3 1	5.25E-07	4 1 3	6.75E-08
1 0 0	1.14E-01	2 3 2	6.76E-08	4 1 4	8.69E-09
1 0 1	1.47E-02	2 3 3	8.71E-09	4 2 0	1.89E-06
1 0 2	1.89E-03	2 3 4	1.12E-09	4 2 1	2.43E-07
1 0 3	2.43E-04	2 4 0	2.44E-07	4 2 2	3.14E-08
1 0 4	3.14E-05	2 4 1	3.14E-08	4 2 3	4.04E-09
1 1 0	6.81E-03	2 4 2	4.05E-09	4 2 4	5.21E-10
1 1 1	8.78E-04	2 4 3	5.22E-10	4 3 0	1.13E-07
1 1 2	1.13E-04	2 4 4	6.72E-11	4 3 1	1.46E-08
1 1 3	1.46E-05	3 0 0	3.16E-03	4 3 2	1.88E-09
1 1 4	1.88E-06	3 0 1	4.07E-04	4 3 3	2.42E-10
1 2 0	4.08E-04	3 0 2	5.25E-05	4 3 4	3.12E-11
1 2 1	5.26E-05	3 0 3	6.76E-06	4 4 0	6.78E-09
1 2 2	6.77E-06	3 0 4	8.71E-07	4 4 1	8.73E-10
1 2 3	8.73E-07	3 1 0	1.89E-04	4 4 2	1.13E-10
1 2 4	1.12E-07	3 1 1	2.44E-05	4 4 3	1.45E-11
1 3 0	2.44E-05	3 1 2	3.14E-06	4 4 4	1.87E-12
1 3 1	3.15E-06	3 1 3	4.05E-07		
				TOTAL	**1.00E+00**

In order to obtain the behaviour of the system by fixing all the routing probabilities and two service rates μ_2 and μ_3 , the following cases with respect to a given parameter are tested. *Case I:-* p_1=0.4, p_2=0.2, p_3=0.5, p_4=0.3, μ_2=1.1, μ_3=2.1.

When input rate λ increases to 0.1 to 0.5 and service rate of first queue μ_1 increases to 0.6 to 1; the average number of couples in the system is calculated by using a Matlab program developed for the purpose. The Table III gives the corresponding values of average number of couples.

Table III: Average number of couples when λ and μ_1 increases

λ	μ_1 =0.6	μ_1 =0.7	μ_1 =0.8	μ_1 =0.9	μ_1 =1.0
0.1	0.4116	0.3783	0.3545	0.3366	0.3227
0.2	0.9833	0.8833	0.8166	0.7690	0.7333
0.3	1.8492	1.5992	1.4492	1.3492	1.2777
0.4	3.3786	2.712	2.3786	2.1786	2.0453
0.5	7.2385	4.7385	3.9052	3.4885	3.2385

The data in Table III is plotted in Figure 2. It gives the relationship between input rate λ and average number of customers in the system. It is clear from the above plot that when input rate λ increases then the average number of couples in the system also increases.

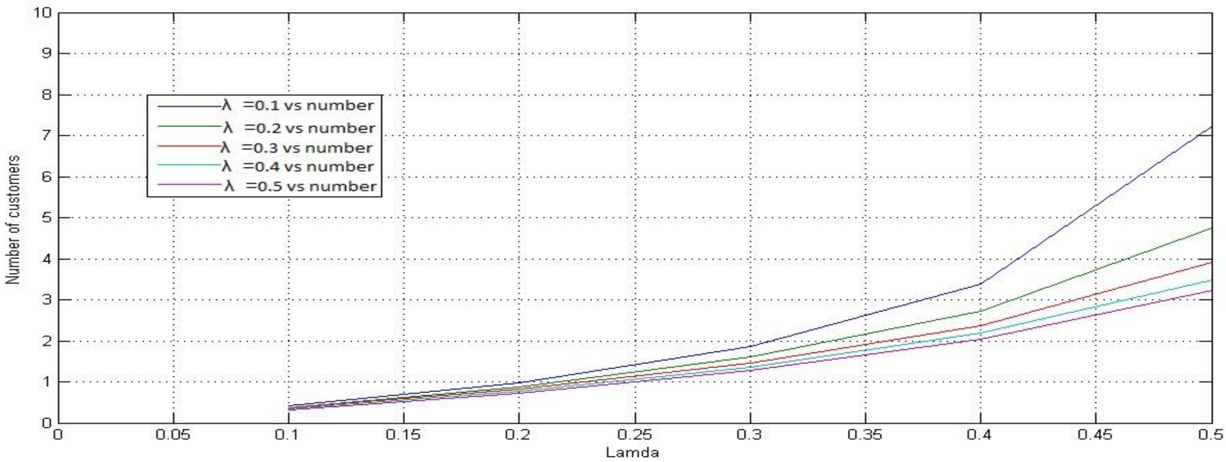

Figure 2 : Relationship between input rate λ and average number of customers

When input rate λ increases to 0.1 to 0.5 and service rate of first queue μ_1 increases to 0.6 to 1; the average waiting time of couples in the system is calculated by using a Matlab program developed for the purpose. The Table IV gives the corresponding values of average waiting time of couples.

Table IV : Average waiting time of couples when λ and μ_1 increases

λ	μ_1 =0.6	μ_1 =0.7	μ_1 =0.8	μ_1 =0.9	μ_1 =1.0
0.1	4.1162	3.7829	3.5448	3.3662	3.2273
0.2	4.9163	4.4163	4.083	3.8449	3.6663
0.3	6.1639	5.3306	4.8306	4.4972	4.2591
0.4	8.4466	6.7799	5.9466	5.4466	5.1132
0.5	14.477	9.477	7.8103	6.9770	6.4770

The data in Table IV is plotted in Figure 3. It gives the relationship between input rate λ and the average waiting time of couples in the system. It is clear from the above plot that when input rate λ increases then the average waiting time of couples in the system also increases.

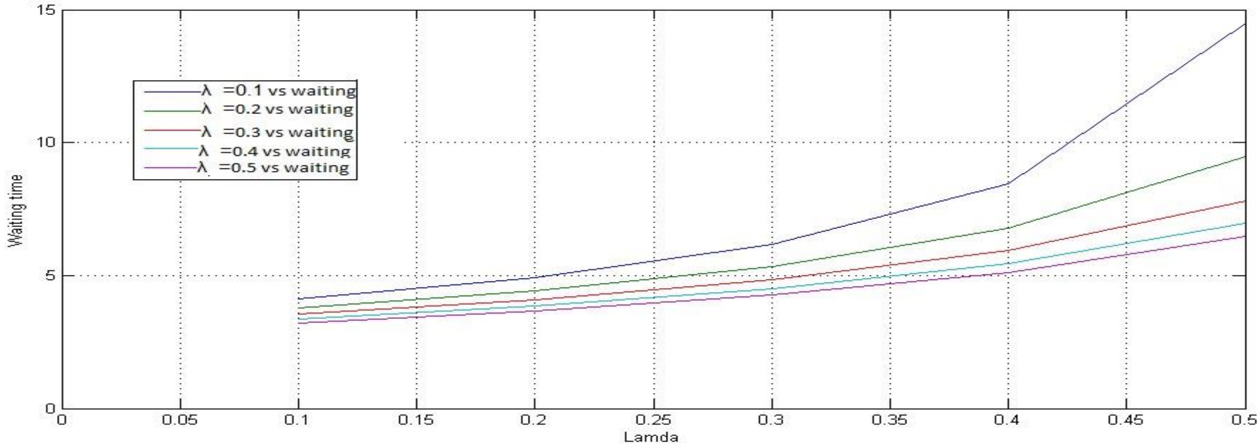

Figure 3 : Relationship between input rate λ and average waiting time of couples

Case II:- p_1=0.4, p_2=0.2, p_3=0.5, p_4=0.3, μ_2=1.1, μ_3=2.1.

When input rate λ increases to 0.1 to 0.5 and service rate of first queue μ_1 increases to 0.6 to 1; the average number of couples in the system is calculated by using a Matlab program developed for the purpose. The Table V gives the corresponding values of average number of couples.

Table V : Average number of couples when μ_1 and λ increases

μ_1	λ =0.1	λ =0.2	λ =0.3	λ=0.4	λ =0.5
0.6	0.4116	0.9833	1.8492	3.3786	7.2385
0.7	0.3783	0.8833	1.5992	2.712	4.7385
0.8	0.3545	0.8166	1.4492	2.3786	3.9052
0.9	0.3366	0.7690	1.3492	2.1786	3.4885
1.0	0.3227	0.7333	1.2777	2.0453	3.2385

The data in Table V is plotted in Figure 4. It gives the relationship between input rate λ and average number of customers in the system. It is clear from the above plot that when service rate of first queue μ_1 increases then the average number of couples in the system decreases.

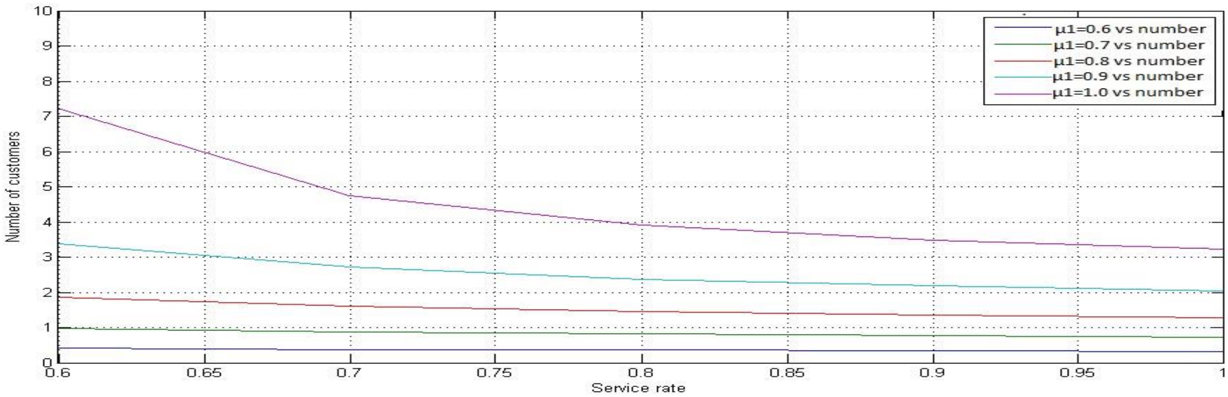

Figure 4 : Relationship between service rate μ_1 and average number of customers

When input rate λ increases to 0.1 to 0.5 and service rate of first queue μ_1 increases to 0.6 to 1; the average waiting time of couples in the system is calculated by using a Matlab program developed for the purpose. The Table VI gives the corresponding values of average waiting time of couples.

Table VI :Average waiting time of couples when μ_1 and λ increases

μ_1	λ =0.1	λ =0.2	λ =0.3	λ=0.4	λ =0.5
0.6	4.1162	4.9163	6.1639	8.4466	14.4770
0.7	3.7829	4.4163	5.3306	6.7799	9.4770
0.8	3.5448	4.083	4.8306	5.9466	7.8103
0.9	3.3662	3.8449	4.4972	5.4466	6.9770
1.0	3.2273	3.6663	4.2591	5.1132	6.4770

The data in Table VI is plotted in Figure 5. It gives the relationship between service rate μ_1 and average waiting time of couples in the system. It is clear from the above plot that when service rate of first queue μ_1 increases then the average waiting time of couples in the system decreases.

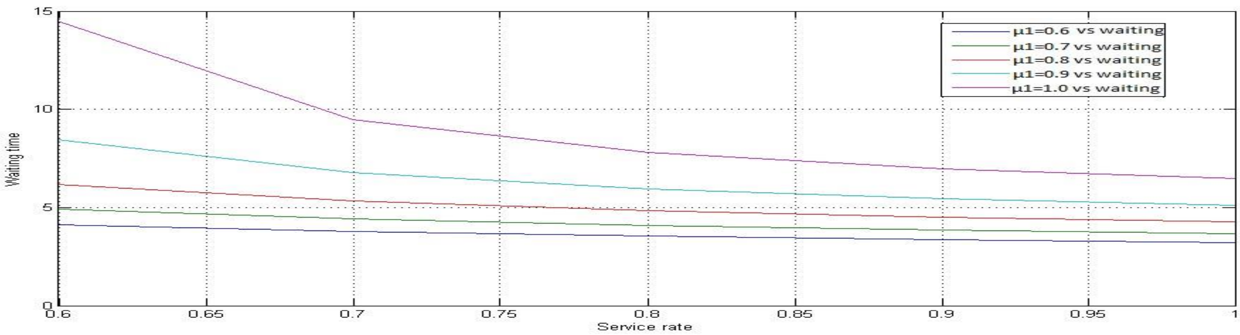

Figure 5 : Relationship between service rate μ_1 and waiting time of couples

VI. CONCLUDING REMARKS

Hospitals and specialized treatment facilities for particular medical conditions such as infertility, cancer, cardiovascular or neurological services, perform a range of services each with its own resources/servers and queues. Such facilities are aptly modeled as networks of queues. In the present work we studied a healthcare system of infertility clinic as an open queueing network with immediate and delayed feedback. Explicit expressions for the steady state behaviour of the system and thereby performance parameters such as average visit count, waiting time, system size etc. are obtained. A Matlab program has been developed to calculate the various parameters of interest concerning the network. Numerical example is presented for illustration. The presented queueing network system offers a good tool for modeling complex service systems. This model is helpful in the preparation of optimal decision on the service organizations and in the study of methods which allow the calculation of basic characteristics of the service process.

It is no surprise that queueing theory has been used extensively in healthcare operations in the light of the various of types of environments and service features in which it can be brought to apply and obtain useful insights for system design and for developing operating principles for service delivery systems. Such models for hospital operations are quite limited and represents an opportunity for future research.

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