FUZZY S- NORMAL AND POLYNOMIAL MATRICES

R.Indira and V.Subharani
[1](Assistant Professor(Sr.Gr),Department of Mathematics, Anna University, CEG campus, Chennai-600025, Tamilnadu)
[2](Research Scholar,Department of Mathematics, Anna University, CEG campus, Chennai-600025, Tamilnadu)

ABSTRACT:

In this paper we introduced the concept of fuzzy s-normal and fuzzy s-normal polynomial matrices and studied some of its properties.

KEYWORDS:

Fuzzy s-normal matrix, Fuzzy s-normal polynomial matrix.

I.INTRODUCTION

Let F = [0,1] be the fuzzy algebra with operations addition and multiplication defined as $a + b = \max \{ a, b\}$ and $ab = \min \{a, b\}$, for every $a, b \in [0,1]$. A fuzzy matrix A of order $n \times n$ is defined as $A = [x_i, a_i]$, where a_i is a membership value of the element x_i in A. For simplicity, we write A as $A = [a_i]$, $a_i \in [0,1]$,i , j = 1 to n [3]. For any matrix $A \in F^{n\times n}$ (set of all fuzzy matrices of order n) , the secondary transpose of A is denoted by A^S and is defined as $A^S = [a_{n-j+1,n-i+1}]$, i , j = 1 to n. $A \in F^{n\times n}$ is said to be s- normal matrix if $AA^S= A^S A$ and is said to be s-unitary matrix if $AA^S = A^S A = I_n$ [1]. Here, we introduce fuzzy s-normal and fuzzy s-normal polynomial matrices and studied some of its algebraic properties. The results are analogous to that of the results studied in [2] for complex case.

II. Fuzzy s- normal matrices

In this section, we present the definition of fuzzy s-normal and fuzzy s-unitary matrices and discuss some of its algebraic properties.

Definition:2.1

Any matrix $A \in F^{n\times n}$ is said to be fuzzy s-normal if $AA^S = A^SA$.

Example:2.2

$A = \begin{bmatrix} 0.4 & 0.2 \\ 0 & 0.3 \end{bmatrix} \in F^{2 \times 2}$ is a fuzzy s-normal matrix.

Theorem:2.3

Let $A \in F^{n\times n}$, then the following conditions are equivalent.

(i) A is fuzzy s-normal matrix.
(ii) A^S is fuzzy s-normal matrix.
(iii) hA is fuzzy s-normal matrix where $h \in F$.

Proof:

(i) $\Leftrightarrow$ (ii):

A is fuzzy s-normal $\Leftrightarrow AA^S = A^S A$
$\Leftrightarrow (A A^S)^S = (A^S A)^S$
$\Leftrightarrow (A^S)^S A^S = A^S (A^S)^S$
$\Leftrightarrow$ A^S is fuzzy s-normal matrix.

(i) $\Leftrightarrow$ (iii):

A is fuzzy s-normal $\Leftrightarrow A A^S = A^S A$.
$\Leftrightarrow h^2 A A^S = h^2 A^S A$.
$\Leftrightarrow (h A) (h A)^S = (h A)^S (h A)$.
$\Leftrightarrow$ hA is fuzzy s - normal matrix.

Theorem:2.4

If A , B $\in F^{n\times n}$ are fuzzy s -normal matrices and $A B^S = B^S A$ and $B A^S = A^S B$, then A + B is fuzzy s-normal matrix.

Proof:

Since A and B are fuzzy s-normal matrices.
We have $A A^S = A^S A$ and $B B^S = B^S B$.
$(A + B) (A + B)^S = AA^S + BB^S + AB^S + B A^S$
$= A^S A+ B^SB + B^SA + A^SB = (A + B)^S (A + B)$.
Hence A + B is fuzzy s-normal matrix.

Theorem :2.5

If A , B $\in F^{n\times n}$ are s-normal matrices and $A B^S = B^S A$ and $B A^S = A^S B$, then AB is also a fuzzy s-normal matrix**.**

Proof:

We have to prove $(AB)(AB)^S = (AB)^S(AB)$
$AB (AB)^S = AB A^SB^S = AA^SBB^S = AA^S B^SB = (BA)^S(AB)$
$= (AB)^S(AB)$.
Hence AB is also a fuzzy s-normal matrix.

Definition :2.6

A matrix A $\in F^{n\times n}$ is said to be s-unitary if $AA^S = A^SA = I_n$. If it satisfies the condition then the possibility of A is a kind of invertible matrix, in particular it will be a permutation matrix.

Theorem:2.7

Let A $\in F^{n\times n}$, then the following conditions are equivalent.
(i) A is fuzzy s-unitary matrix.
(ii) A^S is fuzzy s- unitary matrix.
(iii) hA is fuzzy s-unitary matrix where h$\in$ F .

Proof:

The proof is similar to that of theorem 2.3.

Theorem:2.8

Let A, B $\in F^{n\times n}$. If A and B are fuzzy s-unitary matrices, then AB is a fuzzy s-unitary matrix.

Proof:

A is fuzzy s- unitary then $AA^S = A^SA = I_n$.
B is fuzzy s- unitary then $BB^S = B^SB = I_n$.
$(AB) (AB)^S = A B B^S A^S = A B B^S A^S = A A^S = I_n$.
$(AB)^S (AB) = B^S A^S AB = B^S A^S A B = B^S B = I_n$.
Hence $(AB) (AB)^S = (AB)^S (AB)= I_n$, and AB is fuzzy s-unitary matrix.

III. Fuzzy s-normal polynomial matrices

In this section we define and discuss the fuzzy s-normal and s-unitary polynomial matrices.

Definition:3.1

A fuzzy s-normal polynomial matrix is a polynomial matrix whose coefficient matrices are fuzzy s-normal matrices.

Example :3.2

A(x) be a fuzzy s-normal polynomial matrices.

$$A(x) = \begin{bmatrix} 0.1\,x^2 + 0.2\,x + 0.1 & 0.2\,x^2 + 0.3\,x \\ 0 & 0.1\,x^2 + 0.2\,x + 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix} x^2 + \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix} x + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$= A_2 x^2 + A_1 x + A_0$,where A_0 , A_1, A_2 are fuzzy s-normal matrices.

Theorem:3.3

If $A(\lambda) \in F(\lambda)^{n\times n}$, then the following conditions are equivalent:

(i) $A(\lambda)$ is fuzzy s-normal polynomial matrix.

(ii) $A^S(\lambda)$ is fuzzy s-normal polynomial matrix.

(iii) $hA(\lambda)$ is fuzzy s-normal polynomial matrix, where $h \in F$.

Proof:

The proof is similar lines to that of theorem 2.3, for polynomials.

Theorem :3.4

If $A(\lambda)$, $B(\lambda) \in F(\lambda)^{n\times n}$ (set of all fuzzy polynomial matrices of order n) are s-normal polynomial matrices and $A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$, then $A(\lambda)B(\lambda)$ is also a fuzzy s-normal polynomial matrix**.**

Proof :

Let $A(\lambda) = A_0 + A_1\lambda + \ldots + A_n\lambda^n$ and $B(\lambda) = B_0 + B_1\lambda + \ldots + B_n\lambda^n$ be fuzzy polynomial s-normal matrices, A_0 , $A_1 \ldots A_n$ and B_0 , $B_1 \ldots B_n$ are fuzzy s -normal matrices. Also given ,

$A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$

$A(\lambda)B(\lambda) = A_0 B_0 + (A_0B_1+A_1B_0)\lambda+\ldots+(A_0B_n+A_1B_{n-1}+\ldots+A_nB_0)\lambda^n$

$B(\lambda)A(\lambda) = B_0A_0 + (B_0A_1+B_1A_0)\lambda+\ldots+(B_0A_n+B_1A_{n-1}+\ldots+B_nA_0)\lambda^n$

Here each coefficient of λ and constants terms are equal.

(i.e) $A_0B_0 = B_0A_0$

$A_0B_1 + A_1B_0 = B_0A_1+B_1A_0 \Rightarrow A_0B_1 = B_0A_1$ and $A_1B_0 = B_1A_0$ ....

$A_0B_n+A_1B_{n-1}+\ldots+A_nB_0 = B_0A_n+B_1A_{n-1}+\ldots+B_nA_0$

$\Rightarrow A_nB_0 = B_0A_n , A_1B_{n-1} = B_1A_{n-1} ,\ldots, A_0B_n = B_nA_0$

Now we have to prove $A(\lambda)B(\lambda)$ is s-normal**.**

$A(\lambda)B(\lambda) [A(\lambda)B(\lambda)]^S= A(\lambda)B(\lambda) A^S(\lambda)B^S(\lambda) = A(\lambda)A^S(\lambda)B(\lambda)B^S(\lambda)$

$= A(\lambda)A^S(\lambda) B^S(\lambda)B(\lambda) = [B(\lambda)A(\lambda)]^S[A(\lambda) B(\lambda)] = [A(\lambda) B(\lambda)]^S[A(\lambda) B(\lambda)].$

Hence $A(\lambda) B(\lambda)$ is also a fuzzy s -normal polynomial matrix.

Theorem:3.5

If $A(\lambda)$, $B(\lambda) \in F(\lambda)^{n\times n}$ are fuzzy s-normal polynomial matrices and $A(\lambda) B^S(\lambda) = B^S(\lambda) A(\lambda)$ and $B(\lambda) A^S(\lambda) = A^S(\lambda) B(\lambda)$, then $A(\lambda) + B(\lambda)$ is fuzzy s-normal polynomial matrix.

Proof:

The proof is similar lines to that of theorem 2.4, for polynomials.

Definition :3.6

A fuzzy s-unitary polynomial matrix is a polynomial matrix whose coefficient matrices are fuzzy s-unitary matrices**.**

Theorem:3.7

If $A(\lambda) \in F(\lambda)^{n\times n}$, then the following conditions are equivalent

(i) $A(\lambda)$ is fuzzy s-unitary polynomial matrix.

(ii) $A^S(\lambda)$ is fuzzy s- unitary polynomial matrix.

(iii) $hA(\lambda)$ is fuzzy s-unitary polynomial matrix, where $h \in F$.

Proof:

The proof is similar lines to that of theorem 2.3, for polynomials.

Theorem:3.8

Let $A(\lambda)$, $B(\lambda) \in F(\lambda)^{n\times n}$. If $A(\lambda)$ and $B(\lambda)$ are fuzzy s-unitary polynomial matrices, then $A(\lambda)B(\lambda)$ is fuzzy s-unitary polynomial matrices.

Proof:

The proof is similar lines to that of theorem 2.8, for polynomials.

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