INTUTIONISTIC FUZZY NORMAL POLYNOMIAL MATRICES

R Indira

(Assistant Professor(Sr.Gr),Department of Mathematics, Anna University, CEG campus, Chennai-600025, Tamilnadu)

ABSTRACT:

In this paper we introduce the concept of intutionistic fuzzy normal matrix and intuitionistic fuzzy normal polynomial matrix and studied some of its algebraic properties.

KEYWORDS:

Intuitionistic fuzzy normal matrix, intuitionistic fuzzy normal polynomial matrix

INTRODUCTION:

The fuzzy sets were first introduced by Zadeh[4]. Young Bim and Lee[3] defined the concept of intutionistic fuzzy matrices and Pal, Khan and Shaymal[2] developed some results on intutionistic fuzzy matrices. In this paper, we introduce the definition of intutionistic fuzzy polynomial matrix and intutionistic fuzzy normal polynomial matrix as an extension of fuzzy normal polynomial matrices by Indira and Subharani[1].

Throughout this we consider the matrix over the fuzzy algebra, F = [0,1] with max, min operations.

A fuzzy matrix A of order n × n is defined as A = $[x_{ij}, a_{ij\mu}]$, where $a_{ij\mu}$ is membership value of the element x_{ij} and $a_{ij\mu} \in$ F. An intutionistic fuzzy matrix A of order n × n is defined as A = $[x_{ij}, \langle a_{ij\mu}, a_{ij\lambda}\rangle]$, where $a_{ij\mu}$ is the membership value and $a_{ij\lambda}$ is the non membership value of element x_{ij} in A, which maintain the condition that $0 \leq a_{ij\mu} + a_{ij\lambda} \leq 1$, i, j = 1 to n and $a_{ij\mu}$, $a_{ij\lambda} \in$ F. For simplicity, we write A as A = $[a_{ij}]$, where $a_{ij} = \langle a_{ij\mu}, a_{ij\lambda}\rangle$. For any two element a, b of a matrix A ∈ $F^{n\times n}$(set of all intutionistic fuzzy matrices of order n × n),

$$a + b = \langle \max\{a_{ij\mu}, b_{ij\mu}\}, \min\{a_{ij\lambda}, b_{ij\lambda}\}\rangle \text{ and } a.b = \langle \min\{a_{ij\mu}, b_{ij\mu}\}, \max\{a_{ij\lambda}, b_{ij\lambda}\}\rangle[2].$$

For any matrix A =$[a_{ij}] \in F^{n\times n}$, the transpose of A is denoted by A^T and is defined as $A^T = [a_{ji}]$. A matrix A ∈ $F^{n\times n}$ is said to be symmetric if A = A^T and is said to be normal if $AA^T = A^T A$. An intutionistic fuzzy polynomial matrix is a matrix whose elements are polynomials. For example,

$$A(\lambda) = \begin{bmatrix} \langle 0.2, 0.4\rangle\lambda + \langle 0.3, 0.1\rangle & \langle 0.5, 0.4\rangle\lambda + \langle 0.3, 0.2\rangle \\ \langle 0.3, 0.4\rangle\lambda + \langle 0.2, 0.3\rangle & \langle 0.2, 0.8\rangle\lambda + \langle 0.5, 0.2\rangle \end{bmatrix}$$

is a 2 × 2 intutionistic fuzzy polynomial matrix.

II. Intuitionistic fuzzy normal polynomial matrix

Here, we define intutionistic fuzzy normal polynomial matrix with an example and it is shown that the product of two intutionistic fuzzy normal polynomial matrices is also an intutionistic fuzzy normal polynomial matrix.

Definition:2.1

A intuitionistic fuzzy normal polynomial matrix is a polynomial matrix whose coefficient matrices are intuitionistic fuzzy normal matrices.

Example :2.2

$$A(\lambda) = \begin{bmatrix} \langle 0.2, 0.3\rangle\lambda + \langle 0, 0.1\rangle & \langle 0.1, 0.4\rangle\lambda + \langle 0.2, 0.2\rangle \\ \langle 0, 0.4\rangle\lambda + \langle 0.2, 0.3\rangle & \langle 0.2, 0.3\rangle\lambda + \langle 0.1, 0.2\rangle \end{bmatrix}$$

$$= \lambda \begin{bmatrix} \langle 0.2, 0.3\rangle & \langle 0.1, 0.4\rangle \\ \langle 0, 0.4\rangle & \langle 0.2, 0.3\rangle \end{bmatrix} + \begin{bmatrix} \langle 0, 0.1\rangle & \langle 0.2, 0.2\rangle \\ \langle 0.2, 0.3\rangle & \langle 0.1, 0.2\rangle \end{bmatrix}$$

$= A_1\lambda + A_0$ where A_0 , A_1 are intuitionistic fuzzy normal matrices.

Theorem :2.3

If $A(\lambda)$ and $B(\lambda)$ are intuitionistic fuzzy normal polynomial matrices and $A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$, then $A(\lambda)B(\lambda)$ is also an intuitionistic fuzzy normal polynomial matrix**.**

Proof :

Let $A(\lambda) = A_0 + A_1\lambda + \dots\dots\dots + A_n\lambda^n$ &

$B(\lambda) = B_0 + B_1\lambda + \dots\dots\dots + B_n\lambda^n$ be intuitionistic fuzzy normal polynomial matrix,

A_0 , $A_1 \dots\dots\dots A_n$ and B_0 , $B_1 \dots\dots\dots B_n$ are intuitionistic fuzzy normal matrices and also given ,

$A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$

$A(\lambda)B(\lambda) = (A_0 + A_1\lambda + \dots + A_n\lambda^n)(B_0 + B_1\lambda + \dots + B_n\lambda^n)$

$= (A_0B_0+A_0B_1\lambda+\dots+A_0B_n\lambda^n)+(A_1B_0\lambda+A_1B_1\lambda^2+\dots+A_1B_n\lambda^{n+1})$

$+\dots+ (A_nB_0\lambda^n+ A_nB_1\lambda^{n+1}+\dots+A_nB_n\lambda^{2n})$

$= (A_0+B_0) + (A_0B_1 + A_1B_0)\lambda+\dots+(A_0B_n+A_1B_{n-1}+\dots+A_nB_0)\lambda^n$

$B(\lambda)A(\lambda) = B_0A_0+(B_0A_1+B_1A_0)\lambda+\dots+(B_0A_n+B_1A_{n-1}+\dots+B_nA_0)\lambda^n$

Here each coefficient of λ and constants terms are equal.

(i.e) $A_0B_0 = B_0A_0$

$A_0B_1+A_1B_0 = B_0A_1+B_1A_0$

=> $A_0B_1 = B_0A_1$ & $A_1B_0 = B_1A_0$

…

$A_nB_0 = B_0A_n$, $A_1B_{n-1} = B_1A_{n-1}$,…, $A_0B_n = B_nA_0$

Now we have to prove $A(\lambda)B(\lambda)$ is intuitionistic fuzzy normal matrix**.**

$A(\lambda)B(\lambda) [A(\lambda)B(\lambda)]^T = A(\lambda)B(\lambda)[B(\lambda)A(\lambda)]^T$

$= A(\lambda)B(\lambda) A(\lambda)^TB(\lambda)^T$

$= A(\lambda)A(\lambda)^TB(\lambda)B(\lambda)^T$

$= A(\lambda)^TA(\lambda)B(\lambda)^TB(\lambda)$

$= A(\lambda)^TB(\lambda)^TA(\lambda)B(\lambda)$

$= [B(\lambda)A(\lambda)]^T[A(\lambda) B(\lambda)]$

$= [A(\lambda) B(\lambda)]^T[A(\lambda) B(\lambda)]$

Hence $A(\lambda)\ B(\lambda)$ is intuitionistic fuzzy normal matrix.

Theorem:2.4

Let $A(\lambda)$ be an intutionistic fuzzy normal polynomial matrix of order $n \times n$. Then the following conditions are equivalent:

(i) $A(\lambda)$ is intutionistic fuzzy normal polynomial matrix.

(ii) $A(\lambda)^T$ is intutionistic fuzzy normal polynomial matrix.

Proof:

(i) ⇔ (ii):

$A(\lambda)$ is intutionistic fuzzy normal polynomial $\Leftrightarrow A(\lambda)\, A(\lambda)^T = A(\lambda)^T A(\lambda)$

$\Leftrightarrow (A(\lambda)\ A(\lambda)^T)^T = (A(\lambda)^T A(\lambda))^T$

$\Leftrightarrow (A(\lambda)^T)^T A(\lambda)^T = A(\lambda)^T (A(\lambda)^T)^T$

$\Leftrightarrow A(\lambda)^T$ is an intutionistic fuzzy normal polynomial matrix.

Remark:2.5

Let $A(\lambda)$, $B(\lambda)$ be an intutionistic fuzzy normal polynomial matrices, then its sum is also an intutionistic fuzzy normal polynomial matrix if their product commutes.

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