An Approach to Solve Fully Fuzzy Multi-Objective Linear Programming Problems

Krishnapada Das

Department Of Mathematics, Karimganj College, Karimganj, Assam, India, PIN-788710

Abstract: This paper focuses on the study of fully fuzzy multi-objective linear programming problem (FFMOLPP). Here the cost coefficient, the coefficient matrix and the requirement vector all are characterized by triangular fuzzy numbers. Also the fuzzy variable is considered as triangular fuzzy variable. Applying algebraic operations, the FFMOLPP is converted into multi-objective linear programming problem in crisp environment. Additive weighting method is applied to convert the crisp multi-objective linear programming problem into a single-objective linear programming problem and then by simplex method the problem is solved. The proposed method is illustrated by an example.

Keywords:Fuzzy numbers, Triangular fuzzy number, fully fuzzy multi-objective linear programming problem

1. INTRODUCTION

Linear programming problem is one of the most widely used optimization techniques. It involves a linear function to be optimized (maximization or minimization) with a set of linear constraints. In many real life applications where one may have to optimize a set of linear functions of the same set of decision variables with a set of constraints. In practice the values of the parameters are assigned by the decision makers or experts. However the decision makers and the experts may not be able to assign the exact values of the parameters. In those cases we consider fuzzy data. Bellman and Zadeh [10] first proposed the concept of decision making problems in fuzzy environment.Zimmerman [3] introduced fuzzy linear programming problems with several objective functions. Purnima Pandit [9] discussed fuzzy multi-objective linear programming problems with several objective functions.

In this paper, we propose a method for solving a fully fuzzy multi-objective linear programming problem where the costs/profits, technological coefficients in the constraints and the resources are triangular fuzzy numbers. Also we consider the decision variables as triangular fuzzy variables.

2. PRELIMINARIES

Definition: 1 (Fuzzy Set): Let X be a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0,1]$, which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1]; to each element x of X , where $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$.

Definition: 2 (Support of a fuzzy set): The support of a fuzzy set $\tilde{A}$ on X is the crisp set of those points x of X at which $\mu_{\tilde{A}}(x)$ is positive and is denoted by $Supp(\tilde{A})$, i.e.,

$$Supp(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}.$$

Definition: 3 (Height of a fuzzy set): The height of a fuzzy set $\tilde{A}$ is denoted and defined by $Hgt(\tilde{A}) = Sup\{\mu_{\tilde{A}}(x) : x \in X\}$.

Definition: 4 (Normal fuzzy set): A fuzzy set $\tilde{A}$ on X is called a normal fuzzy set if its height is unity, i.e., $\mu_{\tilde{A}}(x) = 1$, for some $x \in X$. If a fuzzy set is not normal it is called a subnormal fuzzy set.

Definition: 5 (Convex fuzzy set): A fuzzy set $\tilde{A}$ in $X = R^n$ is said to be convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \forall x_1, x_2 \in X, \lambda \in [0,1]$.

Definition: 6 (Fuzzy number): A fuzzy number is a fuzzy set on the real line R which is normal, convex and its membership function is piecewise continuous.

A fuzzy number $\tilde{A}$ is called a positive fuzzy number if $\mu_{\tilde{A}}(x) = 0, \forall x < 0$ and a negative fuzzy number if $\mu_{\tilde{A}}(x) = 0, \forall x > 0$.

Definition: 7 (Triangular fuzzy number): A fuzzy number $\tilde{t}$ denoted by $\tilde{t} = (t, \underline{t}, \bar{t})$ is called a triangular fuzzy number if its member function is given by

$$\mu_{\tilde{t}}(x) = \begin{cases} \dfrac{x - t + \underline{t}}{\underline{t}}, \; if \;\; t - \underline{t} \leq x \leq t \\ \dfrac{t + \bar{t} - x}{\bar{t}}, \; if \;\; t \leq x \leq t + \bar{t} \\ 0, \; otherwise \end{cases}$$

3. Operations on fuzzy numbers defined

Let $\tilde{t}_1 = (t_1, \underline{t_1}, \overline{t_1})$ and $\tilde{t}_2 = (t_2, \underline{t_2}, \overline{t_2})$ be two triangular fuzzy numbers and 'k' is any real number. We define the following operations

$$\tilde{t}_1 + \tilde{t}_2 = (t_1 + t_2, \underline{t_1} + \underline{t_2}, \overline{t_1} + \overline{t_2}),$$
$$\tilde{t}_1 - \tilde{t}_2 = (t_1 - t_2, \underline{t_1} + \underline{t_2}, \overline{t_1} + \overline{t_2})$$
$$k\tilde{t}_1 = (kt_1, |k|\underline{t_1}, |k|\overline{t_1})$$
$$\tilde{t}_1\tilde{t}_2 = (t_1t_2, \underline{t_1t_2}, \overline{t_1t_2})$$

Theorem: Let $\tilde{t}_1 = (t_1, \underline{t_1}, \overline{t_1})$ and $\tilde{t}_2 = (t_2, \underline{t_2}, \overline{t_2})$ be two triangular fuzzy numbers. Then $\tilde{t}_1 = \tilde{t}_2$ if and only if $t_1 = t_2, \underline{t_1} = \underline{t_2}, \overline{t_1} = \overline{t_2}$ and $\tilde{t}_1 \le \tilde{t}_2$ if $t_1 \le t_2, t_1 - \underline{t_1} \le t_2 - \underline{t_2}, t_1 + \overline{t_1} \le t_2 + \overline{t_2}$.

4. FULLY FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEMS (FFMOLPP):

The general form of a FFMOLPP is as follows:

$$\max \tilde{Z}(\tilde{x}) = [\tilde{Z}^1(\tilde{x}),\ \tilde{Z}^2(\tilde{x}), \ldots., \tilde{Z}^k(\tilde{x})]$$
$$s.t.\ \tilde{x} \in \tilde{S} = \left\{\tilde{x} \in \tilde{R}^n : \tilde{A}\tilde{x} \le \tilde{b},\ \tilde{x} \text{ is positive}\right\}$$
$$where\ \tilde{Z}^p(x) = \sum_1^n \tilde{c}_j^p \tilde{x}_j,\ p = 1, 2, \ldots.k$$
$$\tilde{A} = (\tilde{a}_{ij})_{m\times n},\ \tilde{b} = (\tilde{b}_i)$$

The problem after simplification becomes

$$\max \tilde{Z}(\tilde{x}) = [\tilde{Z}^1(\tilde{x}),\ \tilde{Z}^2(\tilde{x}), \ldots., \tilde{Z}^k(\tilde{x})],$$
$$where\ \tilde{Z}^p(\tilde{x}) = \sum_1^n \tilde{c}_j^p \tilde{x}_j,\ p = 1, 2, \ldots k$$
$$s.t. \sum_1^n \tilde{a}_{ij}\tilde{x}_j \le \tilde{b}_i,\ each\ \tilde{x}_j\ is\ positive.$$

Equivalently

$$\max \tilde{Z}(\tilde{x}) = [\tilde{Z}^1(\tilde{x}),\ \tilde{Z}^2(\tilde{x}), \ldots., \tilde{Z}^k(\tilde{x})],$$
$$where\ \tilde{Z}^p(\tilde{x}) = \sum_1^n \tilde{c}_j^p \tilde{x}_j = (\sum_1^n c_j{}^p x_j, \sum_1^n \underline{c_j{}^p x_j}, \sum_1^n \overline{c_j{}^p x_j}),\ p = 1, 2, \ldots k$$
$$s.t. \sum_1^n \tilde{a}_{ij}\tilde{x}_j \le \tilde{b}_i,\ i.e.,\ (\sum_1^n a_{ij}x_j, \sum_1^n \underline{a_{ij}x_j}, \sum_1^n \overline{a_{ij}x_j}) \le (b_i, \underline{b_i}, \overline{b_i}),\ i = 1, 2, \ldots, m.$$
$$And\ each\ x_j\ is\ positive.$$

Now each fuzzy objective function gives rise to three crisp objective functions and as there are k fuzzy objective functions so there are a total of 3k crisp objective functions. Also the theorem on fuzzy inequality stated above gives 3m crisp constraints. At the same time since each of the n fuzzy variable is positive so there are additional n crisp constraints. So the total number of crisp constraints is 3m+n.

Now the FFMOLPP finally becomes

$$\max[Z_1^1, Z_2^1, Z_3^1, Z_1^2, Z_2^2, Z_3^2,,, Z_1^k, Z_2^k, Z_3^k]$$

$$\textit{Where, } Z_1^p = \sum_1^n c_j^p x_j, Z_2^p = \sum_1^n \underline{c_j}^p \underline{x_j}, Z_3^p = \sum_1^n \overline{c_j}^p \overline{x_j},\ 1 \le p \le k$$

subject to

$$\sum_1^n a_{ij} x_j \le b_i,\ \sum_1^n a_{ij} x_j - \sum_1^n \underline{a_{ij}}\,\underline{x_j} \le b_i - \underline{b_i},\ \sum_1^n a_{ij} x_j + \sum_1^n \overline{a_{ij}}\,\overline{x_j} \le b_i + \overline{b_i}$$

$$x_j - \underline{x_j} \ge 0,\ 1 \le i \le m,\ 1 \le j \le n.$$

This is a crisp multi-objective linear programming problem and its efficient solution can be obtained which will give the fuzzy optimal solution to the FFMOLPP. The method precedes by an example.

Let us consider the problem from [9] with fuzzy decision variables.

$$\textit{Find } \tilde{x} = (\tilde{x}_1, \tilde{x}_2) \textit{ such that } \max(\tilde{Z}^1(\tilde{x}) = \tilde{c}_1^1 \tilde{x}_1 + \tilde{c}_2^1 \tilde{x}_2, \tilde{Z}^2(\tilde{x}) = \tilde{c}_1^2 \tilde{x}_1 + \tilde{c}_2^2 \tilde{x}_2)$$

s.t.

$$(3,2,1)\tilde{x}_1 + (6,4,1)\tilde{x}_2 \le (13,5,2)$$

$$(4,1,2)\tilde{x}_1 + (6,5,4))\tilde{x}_2 \le (7,4,2)$$

$\tilde{x}_1$ *and* $\tilde{x}_2$ *are positive.*

where $\tilde{c}_1^1,\ \tilde{c}_2^1,\ \tilde{c}_1^2,\ \tilde{c}_2^2$ *are given by* $(10,3,4),\ (25,5,10),\ (14,4,11),\ (35,10,5)$ *respectively*.

This problem is equivalent to the crisp multi-objective linear programming problem

$$\max\begin{pmatrix} 10x_1 + 25x_2, 3\underline{x_1} + 5\underline{x_2}, 4\overline{x_1} + 10\overline{x_2} \\ 14x_1 + 35x_2, 4\underline{x_1} + 10\underline{x_2}, 11\overline{x_1} + 5\overline{x_2} \end{pmatrix}$$

Subject to the constraints

$$\left.\begin{array}{l} 3x_1+6x_2\le 13 \\ 3x_1+6x_2-2\underline{x_1}-4\underline{x_2}\le 8 \\ 3x_1+6x_2+\overline{x_1}+\overline{x_2}\le 15 \\ 4x_1+6x_2\le 7 \\ 4x_1+6x_2-\underline{x_1}-5\underline{x_2}\le 3 \\ 4x_1+6x_2+2\overline{x_1}+4\overline{x_2}\le 9 \\ x_1-\underline{x_1}\ge 0 \\ x_2-\underline{x_2}\ge 0 \\ x_1,x_2,\underline{x_1},\underline{x_2},\overline{x_1},\overline{x_2}\ge 0 \end{array}\right\}\text{........(1)}$$

Equivalently

$$\max\ w_1(10x_1+25x_2)+w_2(3\underline{x_1}+5\underline{x_2})+w_3(4\overline{x_1}+10\overline{x_2})+w_4(14x_1+35x_2)+w_5(4\underline{x_1}+10\underline{x_2})+w_6(11\overline{x_1}+5\overline{x_2})$$

subject to (1).

This MOLPP is solved by weighting method. The pareto-optimal solutions for different weights is given by the following table.

w_1	w_2	w_3	w_4	w_5	w_6	$(\tilde{x}_1,\tilde{x}_2)$	$\tilde{Z}^1$	$\tilde{Z}^2$
0.3	0.1	0.1	0.3	0.1	0.1	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
0.2	0.2	0.1	0.2	0.2	0.1	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
0.2	0.1	0.2	0.2	0.1	0.2	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
0.2	0.2	0.2	0.2	0.2	0.2	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
0.5	0.5	0.5	0.5	0.5	0.5	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
1.0	1.0	1.0	1.0	1.0	1.0	(0,0,1), (1.16667,1.16667,0)	(29.16675,5.8335,4)	(40.83345,11.6667,11)
0.5	0.0	0.0	0.5	0.0	0.0	(0,0,0), (1.16667,0.8,0)	(29.16675,4,0)	(40.83345,8,0)
0.4	0.1	0.0	0.4	0.1	0.0	(0,0,0), (1.16667,1.16667,0)	(29.16675,5.8335,0)	(40.83345,11.6667,0)
1.0	1.0	0.0	1.0	1.0	0.0	(0,0,0), (1.16667,1.16667,0)	(29.16675,5.8335,0)	(40.83345,11.6667,0)
0.5	0.0	0.0	0.5	0.0	0.0	(0,0,0), (1.16667,0.8,0)	(29.16675,4,0)	(40.83345,8,0)
0.4	0.0	0.1	0.4	0.0	0.1	(0,0,1), (1.16667,0.8,0)	(29.16675,4,4)	(40.83345,8,11)
1.0	0.0	1.0	1.0	0.0	1.0	(0,0,1), (1.16667,0.8,0)	(29.16675,4,4)	(40.83345,8,11)

Conclusion:

In this paper we have presented a method for solving a FFMOLPP with fuzzy variables. The method is illustrated with an example.

References:

1. Amit Kumar, JagdeepKaur, Pushpinder Singh, Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints, International Journal of Mathematical and Computer Sciences, 6, 37-41, (2010).
2. Amit Kumar, JagdeepKaur, Pushpinder Singh, A new method for solving fully fuzzy linear programming problems, Applied Mathematical Modeling, 35, 817-823, (2011).
3. H. J. Zimmerman, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, Vol. 1, 45-55, (1978).
4. H. Tanaka, H. Ichihashi, K. Asai, Formulation of fuzzy linear programming problem based on comparison of fuzzy numbers, Control Cybernetics 3 (3), 185-194, (1991).
5. H. J. Zimmerman, Using fuzzy sets in operations research, EJOR, Vol. 13, 201-216, (1983)
6. H. J. Zimmerman, Optimization in fuzzy environment, Presented at XXI Int. TIMS and 46th ORSA conference, San Juan, Puerto Rico, (1974).
7. L. A. Zadeh, Fuzzy Sets, Information and Control, 8, 338-353, (1965).
8. Masatoshi Sakawa, Hitoshi Yano, Ichiro Nishizaki, Linear and multiobjective programming with fuzzy stochastic extensions, Springer.
9. PurnimaPandit, Multi-objective linear programming problems involving fuzzy parameters, International Journal of Soft Computing and Engineering, Vol. 3(2), (2013).
10. R. E. Bellman, L. A. Zadeh, Decision Making in A Fuzzy Environment, Management Science, vol. 17, 141-164, (1970).