A Parametric Study on the Dynamics of Bi-Articulated Offshore Tower

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ABSTRACT: Offshore compliant structures are a valuable and attractive option for operations in deep water conditions because of their reduced structural weight. One class of compliant structures is the articulated tower which relies on large buoyancy moment to maintain its equilibrium. This paper examines a rigid model of a bi-articulated tower undergoing motion in the plane of fluid loading. Modified Morison equation is used to model the fluid forces. The nonlinear governing equations of motion are derived using Hamilton's principle. Nonlinear effects due to variable submergence, buoyancy, added mass, instantaneous position of the tower and relative-velocity squared drag force are considered in the analysis. The equations of motion are solved numerically in time domain using Wilson-θ method. A parametric study on the tower system is performed, demonstrating the serviceability of the tower in the presence of different tower configurations.

Keywords: articulated tower, compliant tower, dynamic response, offshore

1. INTRODUCTION

An articulated offshore tower as shown in Fig. 1 is a cylindrical structure, having a base establishing a gravity-type foundation on the ocean floor; a ballast chamber, lower shaft, buoyancy chamber, and upper shaft. The part of the structure emerging from the water supports a superstructure arranged to suit a particular application e.g., tanker's mooring and oil loading as well as production riser and control tower. The tower is secured to the base by an articulated joint which permits it to oscillate in any direction. Such conventional tower which is suitable for shallow water depth is called single hinged articulated tower (SHAT).

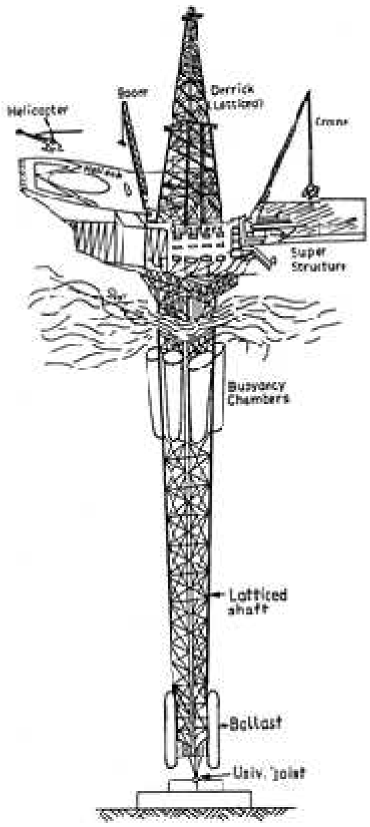

Fig. 1 Single hinged articulated tower

For higher water depths, double hinged articulated towers (DHAT) are used as shown in Fig.2. These towers are made up of two shafts, joined by means of an intermediate hinge. The bottom shaft is connected at the sea bed through another hinge. The upper shaft supports a deck and is equipped with a buoyancy chamber. The tower acquires stability by inherent buoyancy forces against the wind and wave loadings. The structure is assumed to be two degree of freedom system, undergoing motions only in the plane of fluid loading.

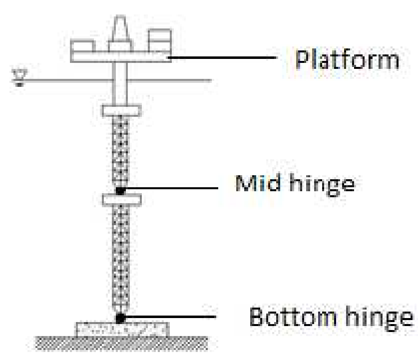

Fig. 2 Double hinged articulated tower

Articulated tower technology was developed in the early of 1970s, and the first articulated tower ever been built was operated in Argyll Field of the North Sea in 1975 [1]. This structure was operated as a single point mooring to a shuttle tanker in conjunction with a semi-submersible as the production unit and a flexible riser. Since then many articulated towers around the world oceans has been constructed and put in operation as the supporting system for offshore hydrocarbon production facility.

2. ARTICULATED TOWER DYNAMICS

The primary feature of an articulated tower is on its ability to displace from its initial position when subjected to environmental loads, and hence reducing the maximum internal reaction on its structural elements. Under the environmental loads, the tower displaces in rotational mode by the virtue of the articulated joint located on the base. In order to revert into its equilibrium position after being displaced, an articulated tower necessitates being equipped with a buoyancy chamber. The buoyancy chamber is considered as one of the most important element since it will provide essential stiffness to the system through the buoyancy restoring forces. The articulated tower is dynamically tuned to have a natural period removed from periods of high wave energy, usually longer than the dominant wave period range. This is accomplished by adjusting the size and location of buoyancy and ballast chambers. In the process of tuning, due consideration is given to axial and horizontal hinge shear, for it is desirable to keep the loads to a minimum from both the foundation and hinge design point of view.

Not much work has been reported on the dynamics of articulated tower. Hanna et al. [2] gave a new concept of Tension Restrained Articulated Platform (TRAP). Study concluded that multi articulation concept is an attractive option for deepwater applications. Sellers and Niedzwecki [3] furnishes valuable inputs on a general mathematical model of multi hinged articulated tower. The model was shown to be useful in changing the natural period of a structure prior to detailed member sizing and weight or buoyancy adjustments.

The dynamic behaviour of articulated tower presented in this paper has been identified under the assumption of rigid body motion in two degree-of-freedom using a certain analytical approach. A particular emphasis is then given on the effect of variations in size, position of the buoyancy chamber and ballast weight towards the response in waves. The current study is expected to form a sensible basis for a further development of sophisticated nonlinear model in higher degrees of freedoms as well as for the investigation of dynamic behaviour on integrated tower and tanker system.

3. BI-ARTICULATED TOWER MODEL

In the present study, a bi-articulated tower is modeled as an inverted double pendulum enacted by two articulation points. A schematic of the tower is shown in Fig. 3. The tower structure is idealized by replacing its mass distribution with discrete masses located at the centroids of a series of small elements. All forces are assumed to act at these centroids and include weight, inertia forces, buoyancy, and fluid forces on the structure. The in plane rotations at the two articulation points constitute the dynamic degree-of-freedom of the system.

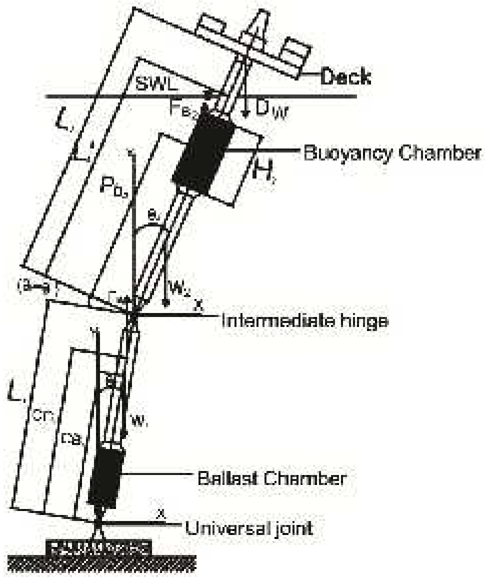

Fig. 3 Schematic of Double hinged articulated tower

4. EQUATIONS OF MOTION

Lagrange's equation can be derived from the principle of virtual displacements or from Hamilton's principle. The latter approach is employed here. This principle occupies an important position in analytical mechanics because it reduces the formulation of a dynamic problem to the variation of two scalar quantities: the work function and the kinetic energy, and because it is invariant under coordinate transformation [4].

According to d'Alembert principle, the virtual work equation is given by

$$\sum_{i=1}^{n}(F_{ai} - m_i \ddot{r}_i)\delta \mathrm{r_i} = 0 \tag{1}$$

Multiplication of (1) by dt and integration between limits t_1 and t_2 gives

$$\int_{t_1}^{t_2}\sum F_i \delta x_i \mathrm{dt} - \int_{t_1}^{t_2}\sum m_i \ddot{x}_i \delta x_i \mathrm{dt} = 0 \tag{2}$$

The first integral on the left hand side of (2) can be written as $\int_{t_1}^{t_2}\overline{\delta W}\mathrm{dt}$, where $\overline{\delta W}$ represents infinitesimal work. The second integral is reduced as

$$-\int_{t_1}^{t_2}\sum m_i \ddot{x}_i \delta x_i \mathrm{dt} = \sum m_i \dot{x}_i \delta x_i\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2}\sum m_i \dot{x}_i \frac{d}{dt}(\delta x_i)\mathrm{dt} \tag{3}$$

It is assumed that the path of all particles is prescribed at time t_1 and t_2, the variations δx_i's are zero at these times and the first term on the right hand side of (3) vanishes giving

$$\int_{t_1}^{t_2}\sum m_i \ddot{x}_i \delta x_i \mathrm{dt} = -\int_{t_1}^{t_2}\sum m_i \dot{x}_i \delta\left(\frac{dx_i}{dt}\right)\mathrm{dt} \tag{4}$$

$$= -\int_{t_1}^{t_2}\delta T \mathrm{dt} \tag{5}$$

where T is the kinetic energy of the system. Substitution of (5) into (2) gives

$$\int_{t_1}^{t_2} \overline{\delta W} + \delta T \mathrm{dt} = 0 \tag{6}$$

Equation (5) is generally known as the extended Hamilton's principle. In the special case when the impressed forces can be derived from a scalar work function, $\overline{\delta W}$ becomes a complete variation and (6) becomes

$$\delta \int_{t_1}^{t_2} (U + T) \mathrm{dt} = 0 \tag{7}$$

where T is the total kinetic energy of the system and V is its potential energy. Recalling that the potential energy function V is the negative of the work function, (7) can be written as

$$\delta \int_{t_1}^{t_2} (T - V) \mathrm{dt} = 0 \tag{8}$$

or

$$\int_{t_1}^{t_2} L \mathrm{dt} = 0 \tag{9}$$

where $L = T - V$ is called the Lagrangian. Hamilton's principle is a variational principle which states that of all possible paths, a mechanical system under motion will take path which makes the integral in (9) a minimum.
For holonomic constraints, the physical coordinates of the system can be expressed in terms of the generalized coordinates and the time variable as

$$x_i = f(q_1, q_2, \ldots.., q_N, t) \tag{10}$$

The Lagrange's equations are obtained from the special form of Hamilton's equations (8) and (9):

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \quad j = 1,2,\ldots., N \tag{11}$$

On setting $L = T - V$ and noting that V does not depend on the velocities, $\dot{q}_j$, (10) can be written in the alternative form as:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0 \quad j = 1,2,\ldots.., N \tag{11}$$

The Lagrange equation for systems subjected to impressed forces can be written as

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} - Q_j = 0 \tag{12}$$

where Q_j represents the generalized force related to q_j generalized coordinate.

The first equation of motion of the tower is given by

$$\left(I_{1t} + m_{2t}L_1^2 + m_d L_1^2\right)\ddot{\theta}_1 - \left[m_{2t}c_2L_1\dot{\theta}_2 \sin(\theta_2 - \theta_1)\right]\dot{\theta}_2 + m_{2t}L_1c_2\cos(\theta_2 - \theta_1)\ddot{\theta}_2 + \left[\{(F_1b_1 - W_1c_1) + (F_2 - W_2 - W_d)L_1\}\frac{\sin\theta_1}{\theta_1}\right]\theta_1 = Q_{\theta_1} \tag{13}$$

Similarly, the second equation of motion is as follows:

$$\left(I_{2t} + I_d + m_d L_p^2\right)\ddot{\theta}_2 + \{m_{2t}c_2L_1\cos(\theta_2 - \theta_1)\}\ddot{\theta}_1 + \{m_{2t}c_2L_1\dot{\theta}_1\dot{\theta}_2 \sin(\theta_2 - \theta_1)\} + \left[(F_2b_2 - W_2c_2 - W_dL_p)\frac{\sin\theta_2}{\theta_2}\right]\theta_2 = Q_{\theta_2} \tag{14}$$

5. NATURAL FREQUENCIES AND MODE SHAPES

The articulated tower is a highly nonlinear system because of nonlinearity in its geometry and the variation in added mass due to the fluctuating sea surface. Hence, it does not have unique frequencies and mode shapes. In realistic sea environment, the frequencies and mode shapes become time variant and change with the change in nonlinearities. One of the approaches to tackle such problem is that the nonlinearities are represented as pseudo forces. The mass and stiffness matrices are calculated based on the initial state, and the corresponding natural frequencies and mode shapes are determined.

For undamped vibration, the equation of motion for the initial state can be written as

$$[M]\{\ddot{x}\}+[K]\{x\}=0 \tag{15}$$

Undamped frequencies and mode shapes of the system are obtained from the above Equation by setting the forcing function to zero. To find the fundamental frequency, the tower response to non-zero initial conditions (small current value) with zero damping is found.

The natural frequencies of the system for the two modes of vibration are obtained as 0.14 rad/sec and 0.42 rad/sec, respectively. The first mode of the system is that in which both upper and lower shafts are displaced in the same direction. This mode is characterized by much lower frequency and higher amplitudes than the second mode, which corresponds to when the two shafts are displaced in the opposite direction.

6. Validation of the Software

Results obtained with the help of the developed program "Double hinged articulated loading platform", (DHALP) are compared with those obtained by Helvacioglu&Incecik [5] for the double articulated towers. Table 1 deal with this validation. The results for the second natural frequency of the tower as obtained by the present study exactly match with the study carried out by Helvacioglu&Incecik [5]. It shows the validity of the mathematicalmodel using rigid body assumption. However, the results of first natural frequency do not match as closely as for second natural frequency. The variation in the numerical values of first natural frequency is mainly due to the basic difference in mathematical model. Thus, the program used for carrying out parametric studies in the present study is validated.

Table 1 Comparison of natural frequencies of articulated tower

Natural frequency	**HelvaciogluIncecik [5]**	**Present study**
First mode	0.10 rad/sec	0.11 rad/sec
Second mode	0.42 rad/sec	0.42 rad/sec

7. NUMERICAL STUDY

In order to demonstrate the structural behaviourof articulated tower a specific example is considered. The length of the bottom and top tower of double hinged articulated tower (DHAT) for the present study are taken as 260 m and 210 m respectively, each with a structural mass of 2.0 E 4 Kg/m. The ballast mass is taken as 44840 Kg/m. The still water depth is taken as 420 m. The effective diameters of the shaft for drag, the buoyancy, and the added mass as well as inertia are 17 m, 7.5 m and 4.5 m, respectively. Likewise, the same effective diameters for the buoyancy chamber are 20 m, 19.5 m and 7.5 m.

Pierson-Moskowitz [6] sea surface elevation spectrum has been used for sea state characterization. Wilson-θ method is employed to solve the equations of motion. Drag and inertia coefficients are taken as 0.6 and 2.0, which are considered as constant all through the water depth. The simulated record length of wave and wind velocity fluctuations is taken for 3600 seconds with a time increment of 0.7 seconds. A uniform current velocity of 1.0 m/sec is considered.

The parametric study is aimed to investigate the effects of variations in tower characteristics on the natural frequencies of the articulated tower. Further, this study is expected to form a sensible basis for a further development of sophisticated nonlinear double articulated tower model under random waves.

The effect of geometrical changes on the natural frequencies of double hinged articulated tower system is given in Figs. 4-8. During this parametric study, only one parameter and its dependent value were changed. The rest of the tower characteristics remained unchanged. The overall trend observed from Figs. 4-8 indicates that the first natural frequency is less sensitive to the changes in the tower characteristics than is the second natural frequency.

Lowering the position of the buoyancy chamber from sea water level (SWL) reduces the second natural frequency (Fig.4). The effect of height of buoyancy chamber on the natural frequencies is depicted in Fig. 5. It shows that boththe natural frequencies are sensitive to increasing the length of buoyancy chamber.

The natural frequencies are more sensitive to the changes in diameter of the buoyancy chamber than to any other geometrical changes (Fig.6). However, it should be noted that the wave force on the buoyancy chamber also increases nonlinearly in proportion to the diameter square. The effect of weight distribution along the bottom shaft (ballast weight) is shown in Fig.7. As is seen from the Figure, the second natural frequency is more sensitive to weight distributions than the first natural frequency. The effect of variation of the deck weight on the natural frequency is also examined. Fig. 8 shows that the naturalfrequency reduction is noticeable if the deck weight increases from 0.25×10^6 Kg to 2.5×10^6 Kg. In order to achieve more stable design configurations, an optimization procedure between the length and diameter of the double hinged articulated tower should be carried out.

From the foregoing discussion, it is obvious that the tower parent configuration is characterized by a very low natural frequency, i.e., in the order of around 0.14 rad/sec or high natural period which is approaching 43.6 sec. Such values indicate that the resonance with most probable sea waves in the operational area was not possible.

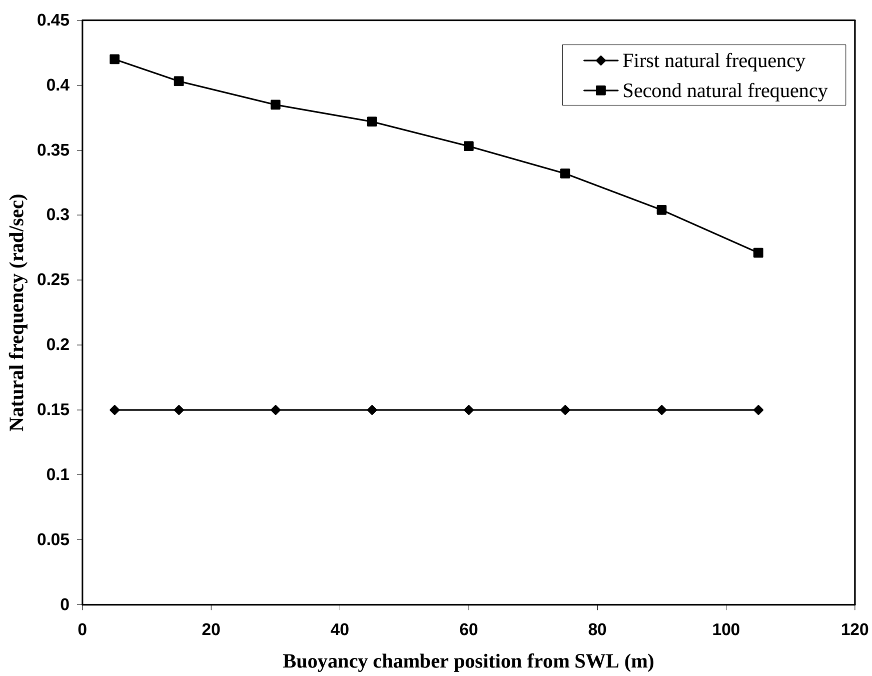

Fig. 4 Sensitivity of natural frequency to position of buoyancy chamber

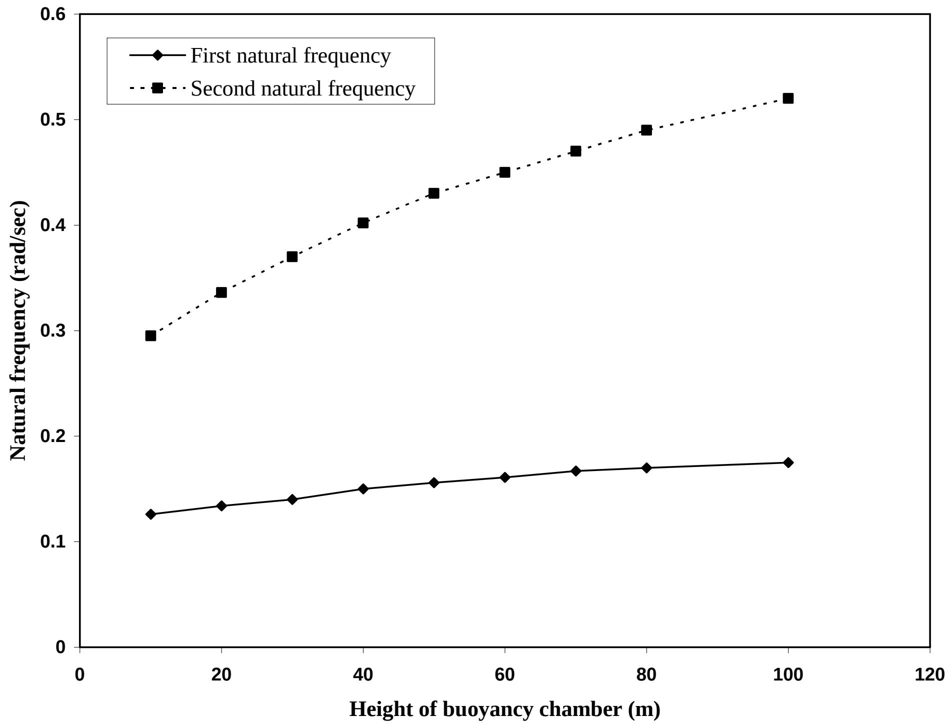

Fig. 5 Sensitivity of natural frequency to height of buoyancy chamber

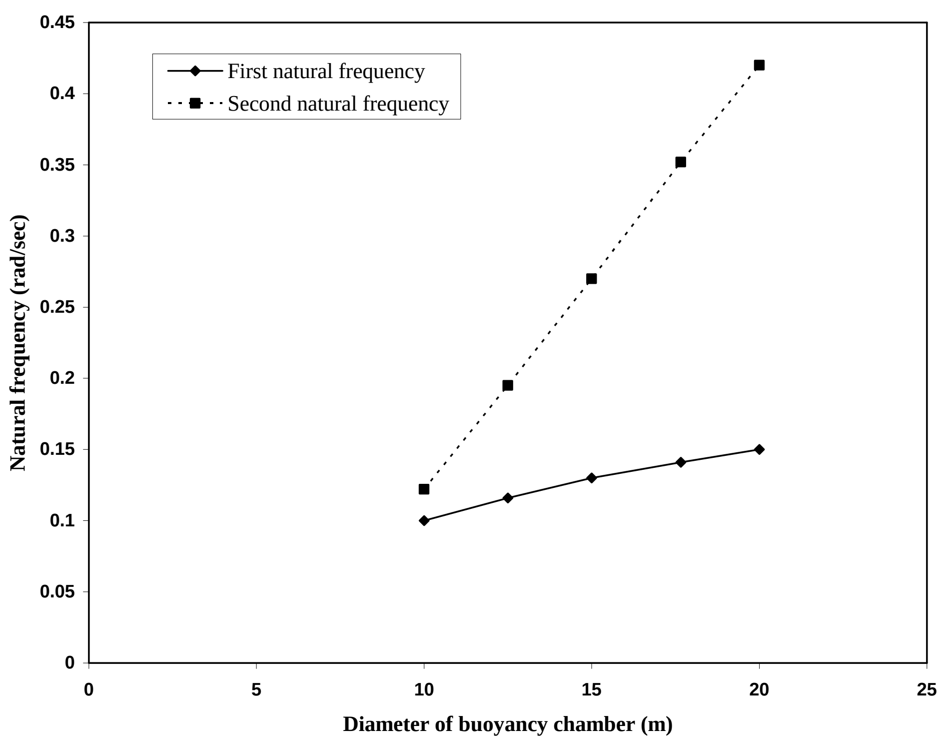

Fig. 6 Sensitivity of natural frequency to diameter of buoyancy chamber

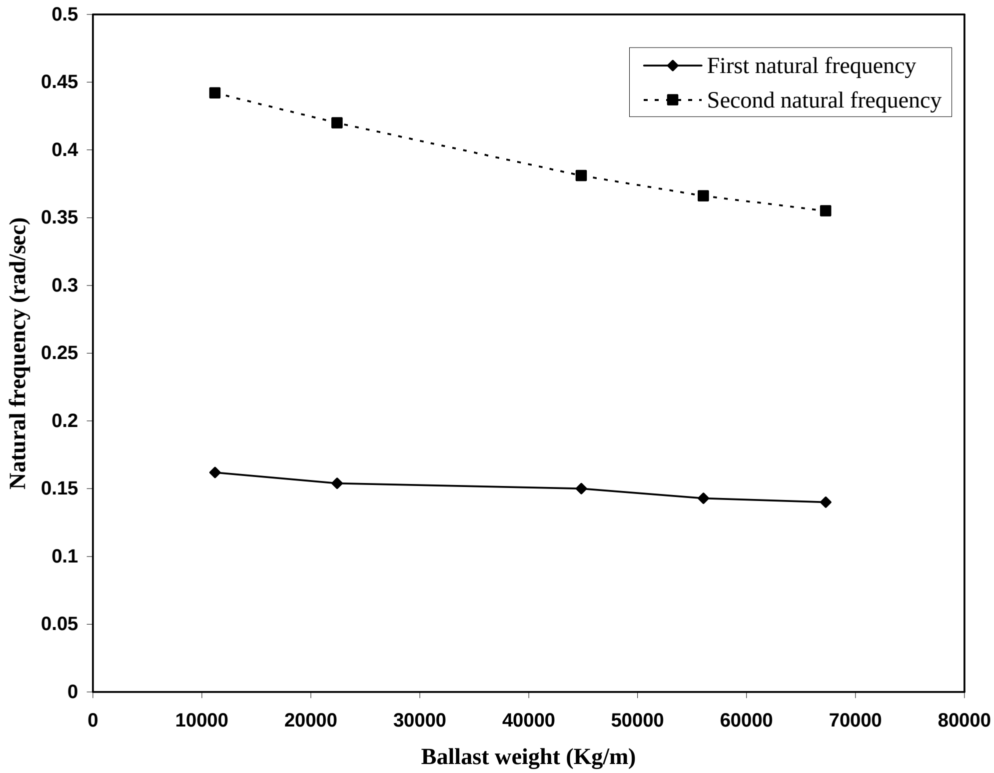

Fig.7 Sensitivity of natural frequency to ballast weight

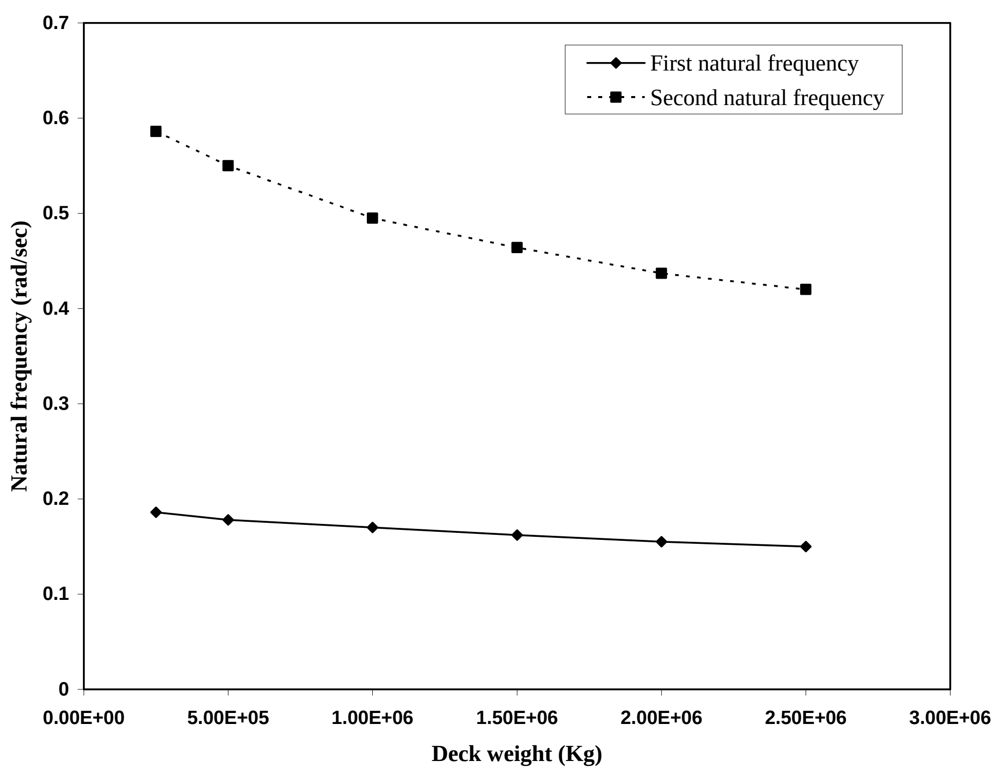

Fig. 8 Sensitivity of natural frequency to deck weight

8. CONCLUSIONS

The mathematical formulation developed in this study is capable of handling nonlinear dynamic analysis by incorporating all the nonlinearities and the solution is stable over a long duration of time. Parametric studies carried out for various buoyancy chamber configurations have shown that variations in the diameter and the depth of buoyancy chamber from SWL have a significant effect on the natural frequencies. The effect of the buoyancy parameter variations is looking quite significant qualitatively, but quantitatively, the effect is almost the same. Based on the outcome, design optimization charts may be prepared for their application in the offshore industry.

9. REFERENCES

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BIOGRAPHY

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